행렬과 3차원 변환

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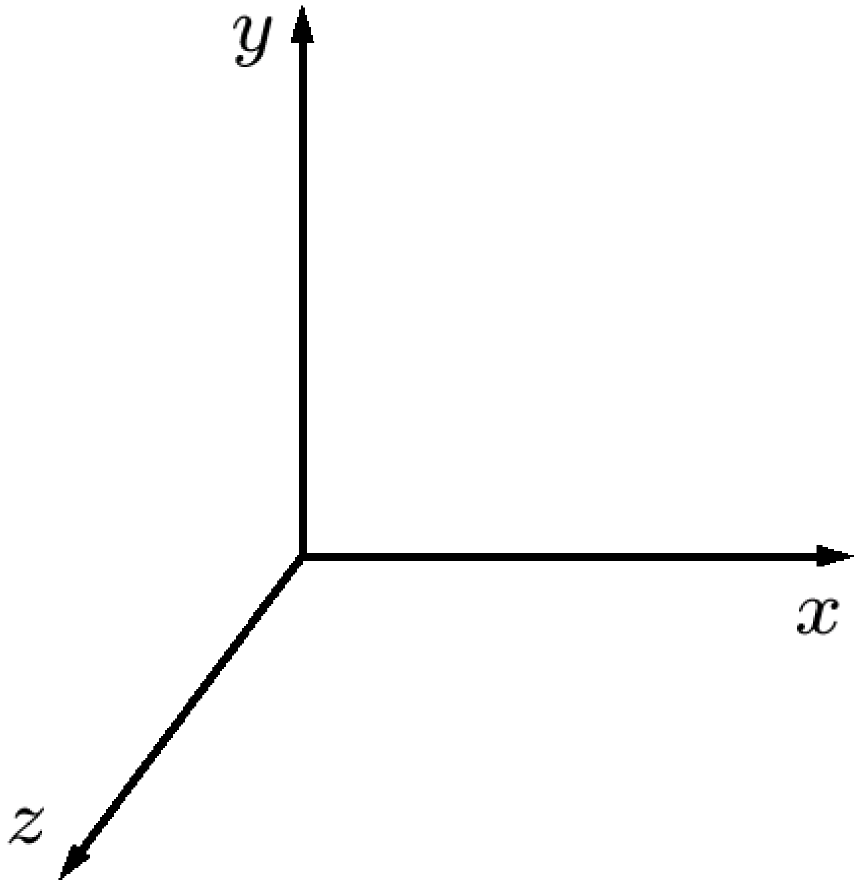
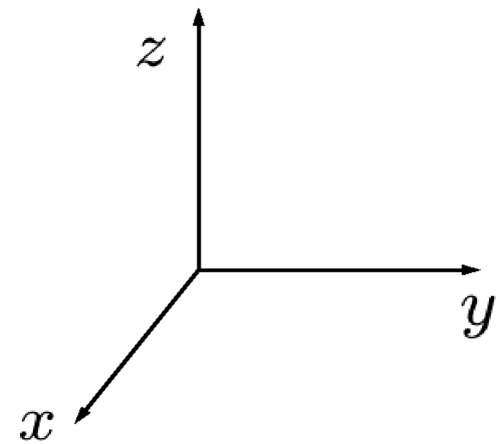
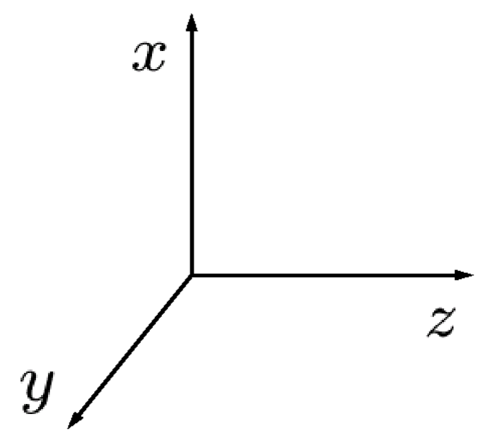
http://3map.snu.ac.kr

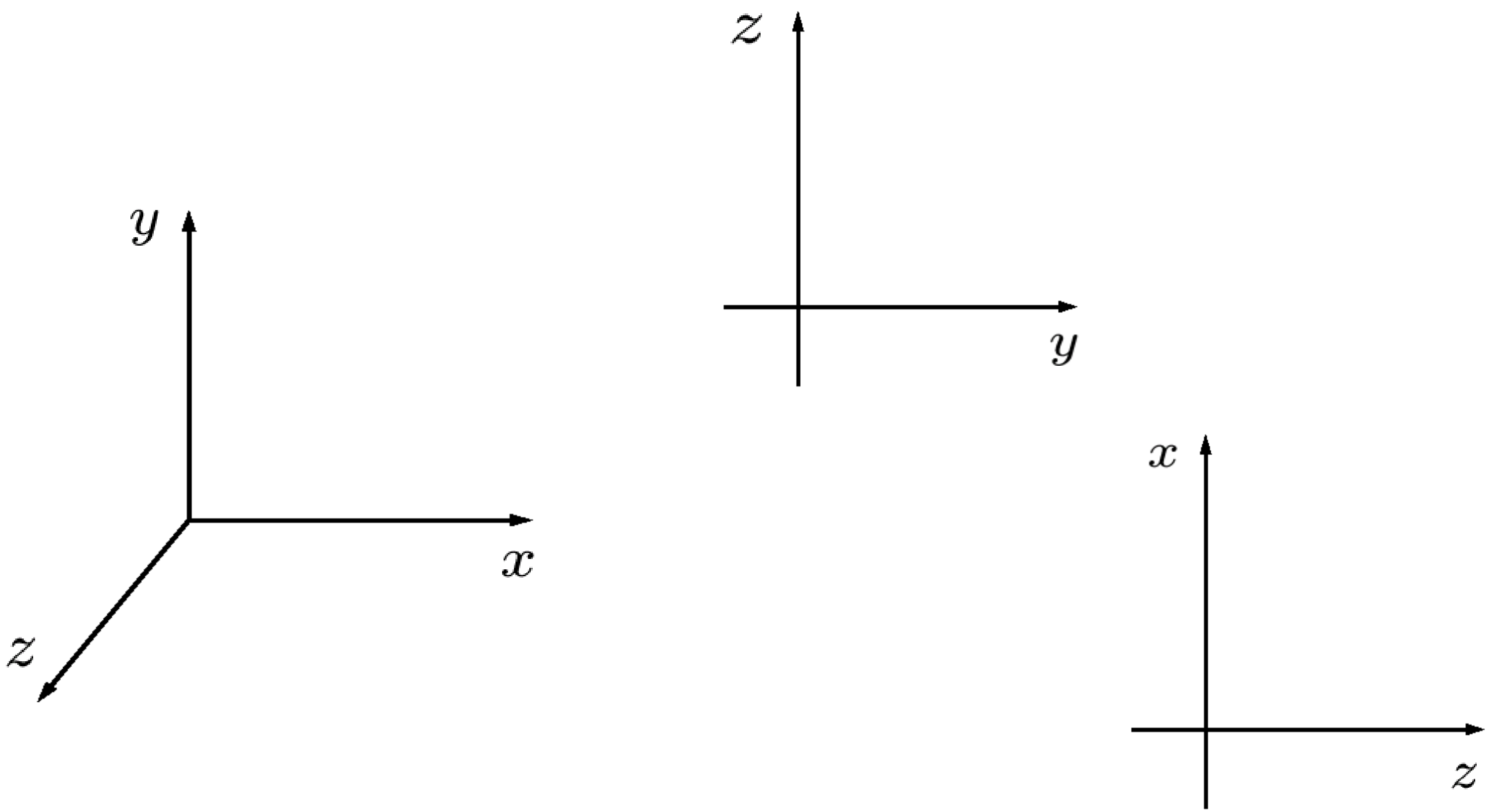
오른손 좌표계





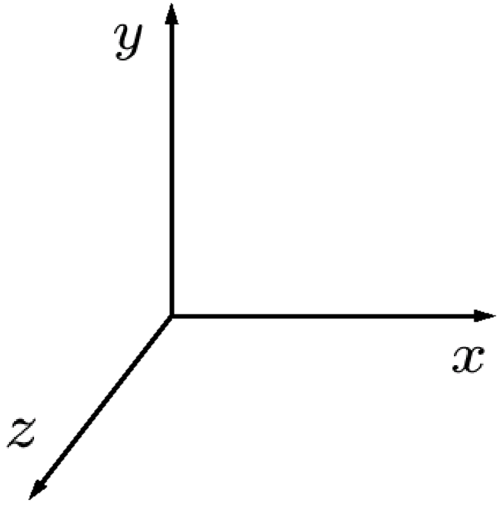


오른손 좌표계



z-축을 중심으로 회전

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



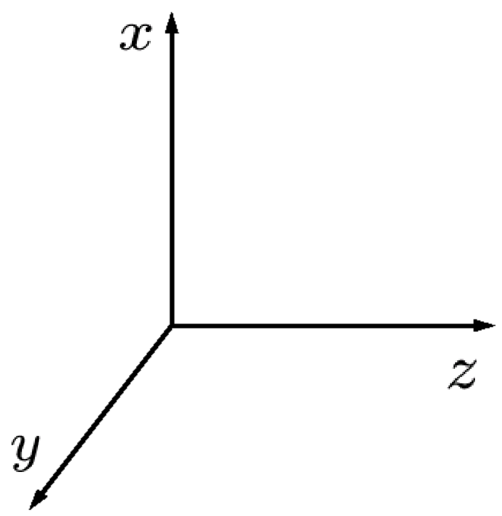
z-축을 중심으로 회전

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \\ 1 \end{bmatrix}$$

y-축을 중심으로 회전

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



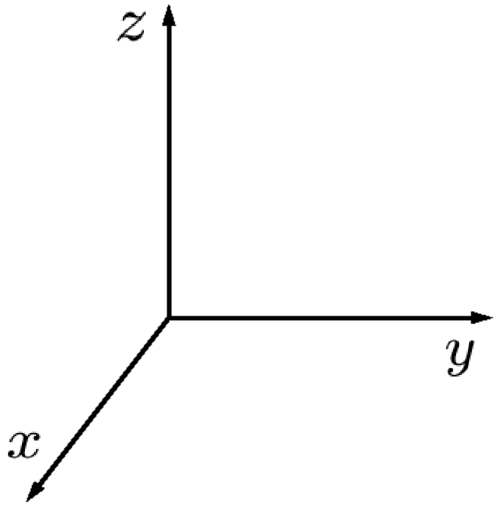
y-축을 중심으로 회전

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ -x\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

x-축을 중심으로 회전

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



x-축을 중심으로 회전

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y\cos\theta - z\sin\theta \\ y\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

3차원 축소확대

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha x \\ \beta y \\ \gamma z \\ 1 \end{bmatrix}$$

3차원 평행이동

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ z + t_z \\ 1 \end{bmatrix}$$

3차원 변환

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ a_z & b_z & c_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

3차원 변환

$$\begin{bmatrix} t_x \\ t_y \\ t_z \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

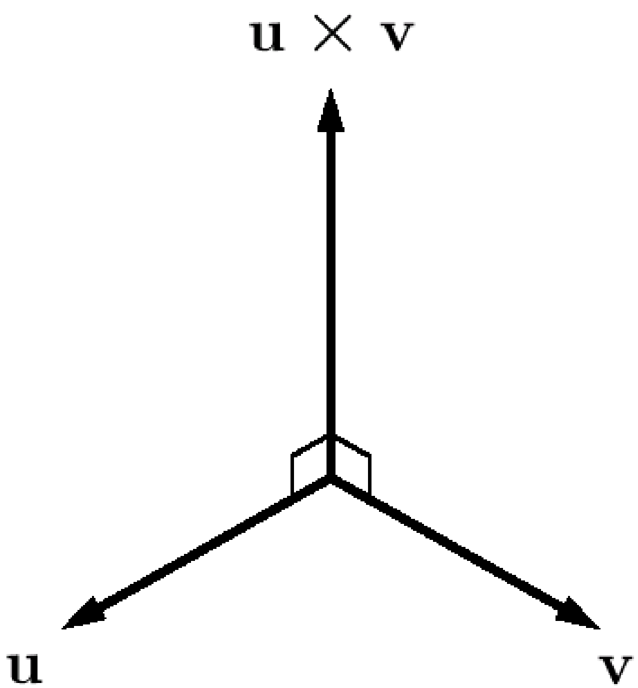
$$\begin{bmatrix} a_x \\ a_y \\ a_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3차원 변환

$$\begin{bmatrix} b_x \\ b_y \\ b_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_x \\ c_y \\ c_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

3차원 벡타의 외적



$$\mathrm{e}_1 = (1, 0, 0)$$
$$\mathrm{e}_2 = (0, 1, 0)$$
$$\mathrm{e}_3 = (0, 0, 1)$$

$$\begin{vmatrix} \mathrm{e}_1 & \mathrm{e}_2 & \mathrm{e}_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= \mathrm{e}_1 \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - \mathrm{e}_2 \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}$$

$$+ \mathrm{e}_3 \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

3차원 벡타의 외적

$$\begin{vmatrix} \mathrm{e}_1 & \mathrm{e}_2 & \mathrm{e}_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= \mathrm{e}_1 \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - \mathrm{e}_2 \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + \mathrm{e}_3 \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$= (u_2 v_3 - u_3 v_2)\mathrm{e}_1 + (u_3 v_1 - u_1 v_3)\mathrm{e}_2$$

$$+ (u_1 v_2 - u_2 v_1)\mathrm{e}_3$$

$$= (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)$$

3차원 벡타의 외적

$$(1,1,0) \times (3,0,0)$$

$$= \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 0 \\ 3 & 0 & 0 \end{vmatrix}$$

$$= e_1 \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} - e_2 \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} + e_3 \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= (-3) \cdot e_3 = (0,0,-3)$$

벡타 외적의 기본성질

$$e_1 \times e_2 = e_3$$

$$e_2 \times e_3 = e_1$$

$$e_3 \times e_1 = e_2$$

$$e_2 \times e_1 = -e_3$$

$$e_3 \times e_2 = -e_1$$

$$e_1 \times e_3 = -e_2$$

벡타 외적의 기본성질

$$(k\mathbf{u}) \times \mathbf{v} = k(\mathbf{u} \times \mathbf{v})$$

$$\mathbf{u} \times (k\mathbf{v}) = k(\mathbf{u} \times \mathbf{v})$$

$$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$$

$$(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times \mathbf{w} + \mathbf{v} \times \mathbf{w}$$

$$\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$$

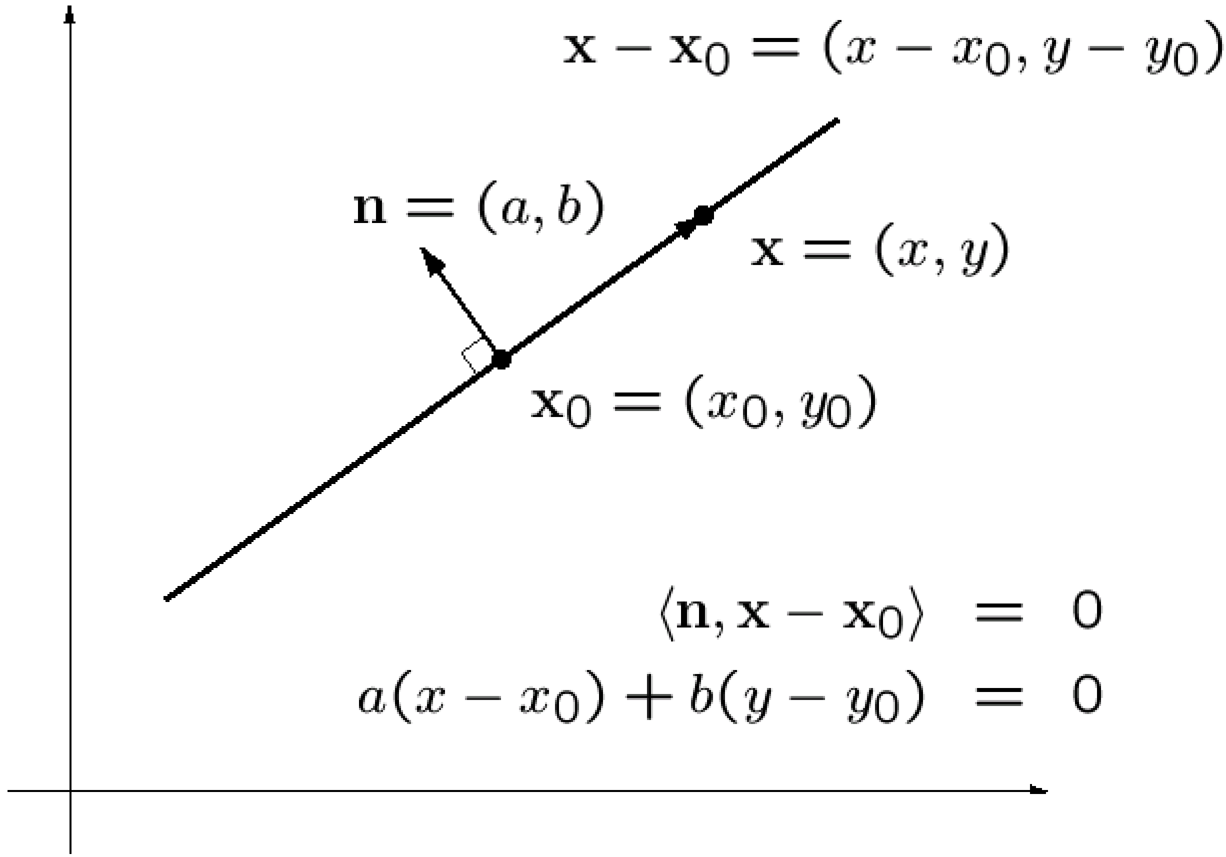
벡타 외적의 기본성질

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) \neq (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \langle \mathbf{u}, \mathbf{w} \rangle \, \mathbf{v} - \langle \mathbf{u}, \mathbf{v} \rangle \, \mathbf{w}$$

$$\langle \mathbf{u}, \mathbf{v} \times \mathbf{w} \rangle = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

직선의 방정식



$$\begin{aligned} \langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle &= 0 \\ a(x - x_0) + b(y - y_0) &= 0 \end{aligned}$$